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Accurate Test of Chiral Dynamics in the $\bar{\gamma}p \rightarrow \pi^0 p$ Reaction

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A precision measurement of the differential cross sections $d\sigma/d\Omega$ and the linearly polarized photon asymmetry $\Sigma \equiv (d\sigma_{\perp} - d\sigma_{\parallel}) / (d\sigma_{\perp} + d\sigma_{\parallel})$ for the $\bar{\gamma}p \rightarrow \pi^0 p$ reaction in the near-threshold region has been performed with a tagged photon beam and almost 4π detector at the Mainz Microtron. The Glasgow-Mainz photon tagging facility along with the Crystal Ball/TAPS multiphoton detector system and a cryogenic liquid hydrogen target were used. These data allowed for a precise determination of the energy dependence of the real parts of the S - and all three P -wave amplitudes for the first time and provide the most stringent test to date of the predictions of chiral perturbation theory and its energy region of agreement with experiment.

Low-energy pion photoproduction experiments are of special interest because the pion, the lightest hadron, is a Nambu-Goldstone boson that by its existence represents a clear signature of spontaneous chiral symmetry breaking in QCD [1]. The dynamic consequences are that the production and elastic scattering of neutral pions at low energies are weak in the S wave and strong in the P wave [1–4], as is seen in the $\gamma N \rightarrow \pi N$ reaction [5, 6]. In neutral-pion photoproduction the S -wave threshold amplitudes are small since they vanish in the chiral limit ($m_u, m_d \rightarrow 0$); their small, but nonvanishing values are consequences of explicit chiral symmetry breaking. In addition, they are isospin violating [2, 3, 7] since $m_u \neq m_d$ [8, 9]. The magnitudes of low-energy scat-

tering and production experiments are predicted by chiral perturbation theory (ChPT), an effective field theory of QCD based on spontaneous chiral symmetry breaking [1–5]. Our efforts have been focused on accurate measurements of low-energy $\gamma N \rightarrow \pi N$ reactions, including the sensitive spin observables that allow a unique separation of the S and P waves, to perform tests of these predictions. As has been stressed [10], any serious discrepancy between these calculations and experiment must be carefully examined as a challenge of our theoretical understanding of spontaneous and explicit chiral symmetry breaking in QCD.

We have conducted an investigation of the $\bar{\gamma}p \rightarrow \pi^0 p$ reaction with the twin goals of obtaining 1) the energy

dependence of the photon asymmetry Σ for the first time and 2) the most accurate measurement to date of the differential cross section from threshold through the Δ region. The energy dependence of Σ , in combination with the cross-section data, allowed us to extract the real parts of all P -wave as well as the S -wave multipoles as a function of photon energy in the threshold region. These data in turn also allowed the first test of how well ChPT calculations agree with the data as a function of photon energy above threshold. There exists one previous measurement of the photon asymmetry [11], but due to poor statistics resulting from small cross sections and limited detector acceptance ($\approx 10\%$ for π^0 detection) Σ was integrated over the entire incident photon energy range, leading to data at only the bremsstrahlung-weighted energy of $E_\gamma = 159.5$ MeV. Moreover, the contribution to the asymmetry from the target walls—significant in the threshold region—was not properly taken into account [12]. With the present setup the azimuthal acceptance was vastly superior *and* symmetric, the degree of linear polarization was higher, a rigorous empty-target subtraction has been done, and as a result both the statistical and systematic uncertainties are much smaller for Σ as well as the cross sections. The most accurate previous measurement along with a list of earlier efforts can be found in Ref. [11].

The experiment that is the focus of this Letter was conducted at the Mainz Microtron MAMI [13, 14] where linearly polarized photons, produced via coherent bremsstrahlung in a 100- μ m-thick diamond radiator [15, 16], were sent through a 4-mm-diameter Pb collimator and impinged on a 10-cm-long liquid hydrogen (LH₂) target located in the center of the Crystal Ball [17]. The TAPS spectrometer served as a forward wall [18], and the LH₂ target was surrounded by a particle identification detector [19], used to differentiate between charged and uncharged particles. The incident photons were tagged up to an energy of 800 MeV using the post-bremsstrahlung electrons detected by the Glasgow-Mainz tagger [20]. For the electron beam of 855 MeV used in this experiment, the tagger channels had a width of about 2.4 MeV in the π^0 threshold region. The diamond radiator was positioned relative to the electron beam such that the photons produced had a polarization in the range 50%–70% between the π^0 threshold and $\simeq 200$ MeV [15].

Neutral pions produced in the LH₂ target were identified in the detector system using their 2γ decay and a kinematic-fitting technique described in detail in Ref. [21]. Both two- and three-cluster events that satisfied the hypothesis of the reaction $\gamma p \rightarrow \pi^0 p \rightarrow \gamma\gamma p$ with a probability of more than 2% were accepted as candidates for this reaction. Background contamination of the event candidates was found to be from two sources: interactions of the bremsstrahlung photons in the target material different from liquid hydrogen, and accidental coincidences between the tagger hits and the trigger based

on the detector signals. The background was subtracted from the signal directly by using two different data samples, the first of which included only events with accidental coincidences, the second taken with an empty-target cell.

Acceptance of the detector system was determined by Monte Carlo simulation of $\gamma p \rightarrow \pi^0 p$ using an isotropic angular distribution. All events were propagated through a GEANT3.21 simulation of the experimental setup, folded with resolutions of the detectors and conditions of the trigger. Close to the reaction threshold, the production-angle acceptance was found to be almost uniform with a detection efficiency about 80%.

The systematic uncertainties in the absolute numbers of the differential cross sections for the reaction $\gamma p \rightarrow \pi^0 p$ obtained in the analysis of the data set were estimated to be not larger than 4%. Such a magnitude of the systematic uncertainty is mostly determined by the calculation of the photon-beam flux, the experimental detection efficiency, and the number of protons in the LH₂ target. The systematic uncertainties in the numbers for the photon asymmetry are on the level of 5%, where this value comes mostly from the uncertainty in the determination of the degree of the linear polarization of the incoming photons.

Results for the differential cross section and the photon asymmetry are presented in Fig. 1 as a function of the pion center-of-mass (c.m.) production angle θ at $E_\gamma = 163.4 \pm 1.2$ MeV, and as a function of incident photon energy at $\theta \simeq 90 \pm 3^\circ$. Also shown are one empirical and two theoretical fits: (1) heavy baryon chiral perturbation theory (HBChPT) calculations to $O(q^4)$ [22] with the five empirical low-energy constants brought up to date by fitting these data [23], (2) relativistic ChPT calculations [also to $O(q^4)$] which as well have five low-energy constants fit to these data [24, 25], and (3) an empirical fit with error bands calculated using the formalism from Refs. [26, 27]. The error bands take into account the correlations among parameters; details on the method can be found in Appendix A of Ref. [23]. Fits have been performed employing a genetic algorithm combined with a gradient-based routine that is thoroughly discussed in Ref. [28].

Because of the high quality of the present data, it is, for the first time, possible to determine the energy range for which ChPT agrees with the data. The values of the low-energy constants were obtained from fits to the data in the range from 150 MeV to a variable E_γ^{\max} . This was done [23] using the $O(q^4)$ formulas of heavy baryon calculations [22] and also for the relativistic theory [24, 25, 29]. Figure 1(e) displays the χ^2 per degree of freedom for the empirical fit and both ChPT calculations. For E_γ^{\max} up to $\simeq 167$ MeV, the ChPT calculations are consistent with the empirical fit, but above this energy the relativistic calculation starts to deviate from the data, whereas the heavy baryon calculation begins to deviate at $\simeq 170$ MeV.

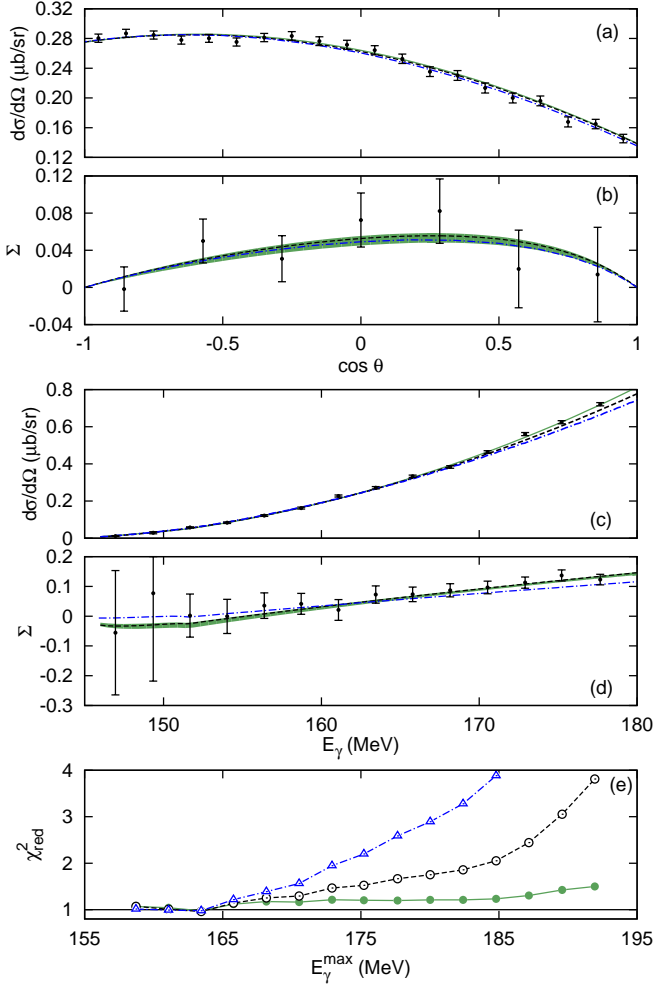


FIG. 1. (color online) Differential cross sections in (a) $\mu\text{b/sr}$ and (b) photon asymmetries for π^0 -production as a function of pion c.m. production angle θ for an incident photon energy of 163.4 ± 1.2 MeV. Energy dependence of the (c) differential cross sections and (d) photon asymmetries at $\theta \simeq 90 \pm 3^\circ$. Errors shown are statistical only, without the systematic uncertainty of 4% for $d\sigma/d\Omega$ and 5% for Σ . The theory curves are dashed (black) for HBChPT [23], dash-dotted (blue) for relativistic ChPT [25, 29], and solid (green) for an empirical fit with an error band. (e) χ^2 per degree of freedom for fits to the data in the range from 150 MeV to E_γ^{max} for HBChPT [23] (open black circles), relativistic ChPT [25, 29] (open blue triangles) and an empirical fit (solid green dots), with lines drawn through the points to guide the eye. Note that in (c) and (d), the two points in incident photon energy below the π^+ threshold are included; these two points are excluded in the fits shown in (e) due to their large error bars.

This is interesting, since the relative contributions of the terms containing the low-energy constants of the relativistic calculation are significantly smaller than those for the heavy baryon version, suggesting a better convergence for the relativistic ChPT method.

The next step in the interpretation of the data is to extract the multipole amplitudes and compare them to the theoretical calculations. To set the notation, the differential cross sections can be expressed in terms of the

S - and P -wave multipoles (E_{0+}, P_1, P_2, P_3) and can be written as

$$\frac{d\sigma}{d\Omega}(\theta) = \frac{q}{k}(A + B \cos \theta + C \cos^2 \theta), \quad (1)$$

where q and k denote the c.m. momenta of the pion and the photon, respectively. The coefficients are given by $A = |E_{0+}|^2 + P_{23}^2$ with $P_{23}^2 = \frac{1}{2}(|P_2|^2 + |P_3|^2)$, $B = 2\text{Re}(E_{0+}P_1^*)$, and $C = |P_1|^2 - P_{23}^2$. The measurement of the cross sections of earlier experiments [11] permitted the extraction of E_{0+} , P_1 and the combination P_{23} . In order to extract the values of $\text{Re}E_{0+}$ and all three P waves separately from the data, it is necessary to also measure the photon asymmetry

$$\Sigma = \frac{d\sigma_\perp - d\sigma_\parallel}{d\sigma_\perp + d\sigma_\parallel} = \frac{q}{2k}(|P_3|^2 - |P_2|^2) \sin^2 \theta / \frac{d\sigma}{d\Omega}(\theta), \quad (2)$$

where $d\sigma_\perp$ and $d\sigma_\parallel$ are the differential cross sections for photon polarization perpendicular and parallel to the reaction plane with the pion and the outgoing proton. To reiterate, the measurement of the differential cross section and Σ allows for the separation of the four multipoles. It is important to note that the determination reported here is more accurate than previous ones due to the far smaller uncertainties of the cross sections as well as the energy dependence of Σ . Furthermore, we note that the D waves have been neglected in both (1) and (2), but they have recently been shown to be important in the near-threshold region [30]. Since there are insufficient data to determine the D -wave multipoles empirically, they have been taken into account by using their values in the Born approximation, which is sufficiently accurate for the present analysis.

The empirical fits to the data employ the following ansatz for the S - and P -wave multipoles

$$E_{0+}(W) = E_{0+}^{(0)} + E_{0+}^{(1)} \left(\frac{E_\gamma - E_\gamma^{\text{thr}}}{m_{\pi^+}} \right) + i\beta \frac{q_{\pi^+}}{m_{\pi^+}}, \quad (3)$$

$$P_i(W) = \frac{q}{m_{\pi^+}} \left[P_i^{(0)} + P_i^{(1)} \left(\frac{E_\gamma - E_\gamma^{\text{thr}}}{m_{\pi^+}} \right) \right], \quad (4)$$

where here E_γ and E_γ^{thr} are in the lab frame, and $E_{0+}^{(0)}, E_{0+}^{(1)}, P_i^{(0)}, P_i^{(1)}$ (with $i = 1, 2, 3$) are constants that are fit to the data. [The empirical values are in units of $10^{-3}/m_{\pi^+}$: $E_{0+}^{(0)} = -0.369 \pm 0.027$, $E_{0+}^{(1)} = -1.47 \pm 0.13$, $P_1^{(0)} = 9.806 \pm 0.068$, $P_1^{(1)} = 1.63 \pm 0.32$, $P_2^{(0)} = -10.673 \pm 0.070$, $P_2^{(1)} = -4.52 \pm 0.31$, $P_3^{(0)} = 9.671 \pm 0.060$, $P_3^{(1)} = 15.87 \pm 0.29$. The pairs $(E_{0+}^{(0)}, E_{0+}^{(1)})$ and $(P_i^{(0)}, P_i^{(1)})$, $i = 1, 2, 3$, are highly correlated.]

Based on unitarity, the cusp parameter in Eq. (3) has the value $\beta = m_{\pi^+} a_{\text{cex}}(\pi^+ n \rightarrow \pi^0 p) \text{Re}E_{0+}(\gamma p \rightarrow \pi^+ n)$ [31]. Using the experimental value of $a_{\text{cex}}(\pi^- p \rightarrow$

$\pi^0 n) = -(0.122 \pm 0.002)/m_{\pi^+}$ obtained from the observed width in the $1s$ state of pionic hydrogen [32], assuming isospin is a good symmetry, i.e. $a_{\text{cex}}(\pi^+ n \rightarrow \pi^0 p) = -a_{\text{cex}}(\pi^- p \rightarrow \pi^0 n)$, and the latest measurement for $E_{0+}(\gamma p \rightarrow \pi^+ n) = (28.06 \pm 0.27 \pm 0.45) \times 10^{-3}/m_{\pi^+}$ [33], we obtain $\beta = (3.43 \pm 0.08) \times 10^{-3}/m_{\pi^+}$, which was employed in the empirical fit. If isospin breaking is taken into account [34, 35] we obtain $\beta = (3.35 \pm 0.08) \times 10^{-3}/m_{\pi^+}$. In this experiment we do not have access to the imaginary part of the S -wave amplitude and no difference is found if either the isospin-conserving, the isospin-breaking, or even other β values such as those for dispersive effective chiral theory, $\beta = 3.10 \times 10^{-3}/m_{\pi^+}$ [36] or HBC χ PT $\beta = 2.72 \times 10^{-3}/m_{\pi^+}$, are employed. Hence, the uncertainty introduced by the errors in β and isospin breaking is smaller than the statistical uncertainties of the multipole extraction depicted in Fig. 2.

The extracted multipoles are displayed in Fig. 2 along with the theoretical calculations. The points are single-energy fits to the real parts of the S - and P -wave multipoles, and the energy-dependent fits from Eqs. (3) and (4) are shown with the error band. The imaginary part of the S -wave multipole E_{0+} was taken from unitarity (3) with the value of the cusp parameter explained above, the imaginary parts of the P waves were assumed to be negligible, and the D -wave multipoles were calculated in the Born approximation. The impact of D waves in the P -wave extraction is negligible [30] but in the S wave it can be sizeable. In order to assess the uncertainties in the S -wave extraction associated to our D -wave prescription, we have estimated the uncertainty from the difference between the Born terms and the Dubna-Mainz-Tapei dynamical model in Ref. [37]. This error estimation is depicted in Fig. 2 as a gray area at the top of the first plot. Note that the D waves have a negligible impact in the P -wave extraction (the uncertainty is smaller than the curve's width) [30].

As was the case for the observables, there is very good agreement between the two ChPT calculations and the empirical values of the multipoles for energies up to $\simeq 170$ MeV with the same pattern of deviations above that.

In conclusion, the combination of the photon asymmetry and improved accuracy in the differential cross section has allowed us to extract the real parts of the S -wave and all three P -wave multipoles as a function of photon energy for the first time. We have achieved an unprecedented accuracy in our empirical extraction of the multipoles from the data, providing a more sensitive test of the ChPT calculations than has previously been possible. What we have found is that none of the real parts of the multipoles E_{0+} , P_1 , P_2 , P_3 is causing the gradual deviation from experiment (increasing χ^2) with increasing energy. Rather, it is probably due to the gradually increasing importance of the higher-order terms neglected in the chiral series, or to the fact that the Δ degree of

freedom is not being taken into account in a dynamic way.

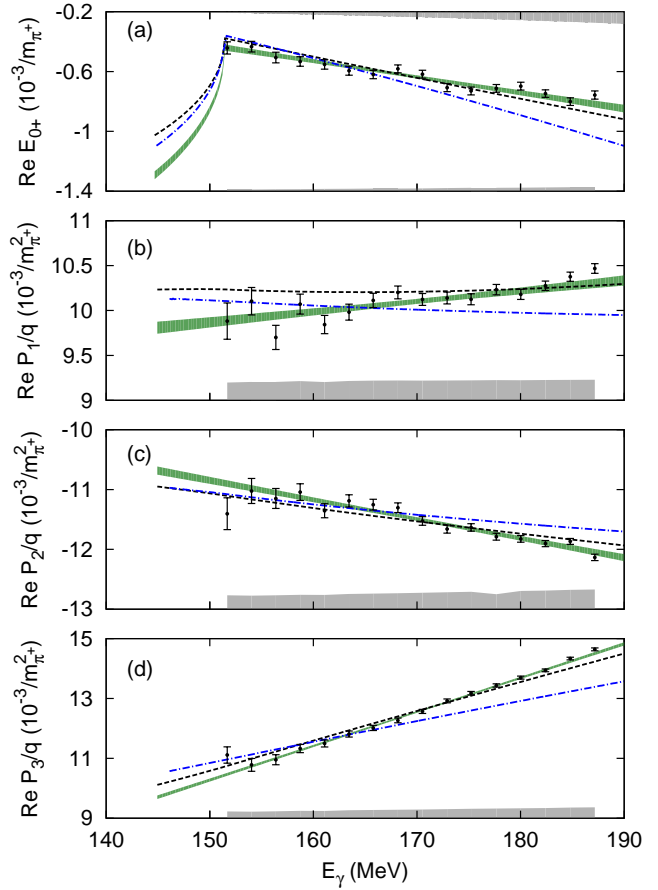


FIG. 2. (color online) Empirical multipoles as a function of incident photon energy: (a) $\text{Re}E_{0+}$, (b) $\text{Re}P_1/q$, (c) $\text{Re}P_2/q$, (d) $\text{Re}P_3/q$. The points are single-energy fits to the real parts of the S - and P -wave multipoles, and the empirical fits from Eqs. (3) and (4) are shown with (green) statistical error bands. The \pm systematic uncertainty for the single-energy extraction is represented as the gray area above the energy axis, and the systematic uncertainty in the S -wave extraction due to the uncertainty in the size of the D -wave contributions is given by the gray area at the top of (a). The theory curves are the same as in Fig. 1. Note that the two points in incident photon energy below the π^+ threshold are excluded from all fits due to their disproportionately large error bars.

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